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Walter J. Heikkila

Southwest Center for Advanced Studies
P. O. Box 8478
Dallas 5, Texas

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A RADIO FREQUENCY PROBE FOR ELECTRON DENSITY MEASUREMENTS

W. J. Heikkila
Southwest Center for Advanced Studies
P. O. Box 8478
Dallas 5, Texas

1. Introduction

A radio frequency probe for electron density measurements in the D and E regions of the ionosphere has been developed at the Defence Research Telecommunications Establishment, Ottawa, Canada, and used successfully on two rocket flights in May and June, 1963. The sensing element of the probe consists of the capacitor formed by the 25 inch long nose tip of the rocket, which is separated from the rest of the rocket by a 5 inch fiberglass spacer. This capacitor forms a major part of the capacity in the tank circuit of a high frequency oscillator. When the tip is immersed in a plasma, the effective capacity is reduced and the oscillator frequency is raised. From the measured increase in frequency, it is possible to deduce the real part of the dielectric constant of the medium, and hence the electron density if the collision frequency is known or assumed. By the use of two slightly differing oscillator frequencies it is possible to estimate the electron collision frequency as well as the electron density.

2. Simple circuit theory of the probe

Consider two conducting spheres, one of radius R_o representing the rocket or satellite vehicle in space, and the other of smaller radius R representing the probe held at a fixed distance from the vehicle (Fig. 1). Let the probe be connected by an insulated wire to an inductance L inside the main vehicle. The spherical probe acts as a condensor of capacitance $C_p = 4\pi\varepsilon\varepsilon_o R$ farads when it is infinitely far from other bodies and embedded in a uniform medium with dielectric constant ε (MKS units). Neglecting for the moment the effect of other capacities, the probe capacity in series with the inductance will resonate at a frequency f given by

$$f = \frac{1}{2\pi\sqrt{LC_p}} \tag{1}$$

$$= \frac{1}{2\pi\sqrt{LC_{po}}} \cdot \frac{1}{\sqrt{\epsilon}}$$

$$\text{or} \quad f = \frac{f_o}{\sqrt{\epsilon}} \tag{2}$$

where ε is the dielectric constant and the subscript o indicates values of the capacity and resonant frequency in free space when $\varepsilon = 1$. The device thus constitutes a probe for the measurement of the dielectric constant of the medium in which the probe is immersed.

In practice, the probe capacity will not be the only capacity in the circuit. The probe itself (C_p) is effectively in series with the space vehicle (C_v). Furthermore, stray capacities C_c associated with the leads, the inductance, and the oscillator are effectively in parallel with the combination as shown in Fig. 1b. Resistance elements R_p and R_v have been included to show the possible presence of losses in the dielectric medium; these may affect the resonant frequency if the oscillator does not seek the zero phase shift point, but will be neglected here. Since C_v will in general be large compared to C_p, its effect in series with C_p is small. In practice, the stray-capacity, C_c, may be comparable to C_p and must be taken into account. The combined result of these other elements is to reduce the frequency dependence on the external dielectric constant; it will be assumed in the following that C_c is the only significant element in the circuit other than C_p.

The relative frequency change $\frac{f - f_o}{f}$ under these assumptions is as follows.

$$f = \frac{1}{2\pi\sqrt{L(C_c + \epsilon C_p)}}$$

$$f_o = \frac{1}{2\pi\sqrt{L(C_c + C_p)}}$$

$$\frac{f - f_o}{f_o} = \sqrt{\frac{C_c + C_p}{C_c + \epsilon C_p}} - 1$$

$$= \frac{1 - \epsilon}{2} \cdot \frac{C_p}{C_c + C_p} \qquad (1-\epsilon) \text{ small}$$

or

$$\frac{f - f_o}{f_o} = \frac{K}{2}(1 - \epsilon) \tag{3}$$

This is the probe equation for small departures of ε from 1. The factor 1/2 arises from the square root dependence of frequency on capacity; the factor K, equal to the ratio of the probe capacity that is affected by the external medium to the total circuit capacity, represents the approach to an ideal circuit with no stray capacities for which K = 1. For the DRTE probe, K ≈ 1/2; it may be evaluated by a calibration procedure in which the frequency change for a known capacity change is measured.

3. <u>Electron Density Measurement</u>

In a plasma in which electron collisions and the magnetic field may be neglected, the dielectric constant ε at a frequency f is given in terms of the electron density N, charge e and mass m by

$$\epsilon = 1 - \frac{e^2 N}{4\pi^2 \epsilon_o m f^2} \tag{4}$$

$$\text{or} \qquad \epsilon = 1 - \frac{81\, N}{f^2} \tag{5}$$

Where N is the electron density per cubic meter, and f is in cps. When the operating frequency f is much greater than the plasma frequency, $f_c = 81N = \frac{\omega_c}{2\pi}$, and the probe equation is

$$\frac{f - f_o}{f_o} = \frac{81\, K N}{2 f^2} = \frac{K}{2} \cdot \frac{\omega_c^2}{\omega^2}, \quad \omega_c^2 \ll \omega^2 \tag{6}$$

For very small electron densities $f \approx f_o$, the change in operating frequency is directly proportional to the electron density in the plasma.

$$f - f_o = \text{const.} \times N \tag{7}$$

This simple linear dependence is followed closely as long as the oscillating frequency is several times the plasma frequency.

4. Electron Collision Frequency Measurement

When the electron collision frequency ν is appreciable the real part of the dielectric constant must be used in the probe equation. According to the Appleton-Hartree theory it is given by

$$\varepsilon_r = 1 - \frac{\omega_c^2}{\nu^2 + \omega^2} \tag{8}$$

The probe equation then becomes

$$\frac{f - f_o}{f_o} = \frac{K}{2} \cdot \frac{\omega_c^2}{\nu^2 + \omega^2} \tag{9}$$

In this case the frequency change is reduced by an amount that depends on the operating frequency. This dependence provides the possibility of evaluating the collision frequency as well as the electron density by the use of two probes with different frequencies f_1 and f_2; then the ratio R of the relative frequency changes is given by

$$\mathcal{R} = \left(\frac{f_1 - f_{10}}{f_{10}}\right) \Big/ \left(\frac{f_2 - f_{20}}{f_{20}}\right) = \frac{\nu^2 + \omega_2^2}{\nu^2 + \omega_1^2} \tag{10}$$

$$= \frac{\nu^2 + 4\pi^2 f_2^2}{\nu^2 + 4\pi^2 f_1^2} \tag{11}$$

It is now generally accepted that the effect of electron collisions is not satisfactorily taken into account in the Appleton-Hartree theory, in which ν is assumed to be independent of electron velocity. The generalized theory, as presented by Sen and Wyller (1960) and other, permits taking a velocity dependence of ν into account. Observations have indicated that ν is proportional to the electron energy in air, $\nu = \nu_m \frac{mv^2}{2kT}$. Again neglecting the effect of the steady magnetic field, equation (56) of Sen and Wyller's paper gives

$$1 - \epsilon_r = \frac{\omega_c^2}{\nu_m^2} \mathcal{C}_{3/2}\left(\frac{\omega}{\nu_m}\right) \tag{12}$$

where

$$\mathcal{C}_{3/2}(x) = \frac{1}{3/2!} \int_0^\infty \frac{t^{3/2} e^{-t}}{t^2 + x^2} \, dt \tag{13}$$

has been tabulated (Dingle, Arndt, and Roy, 1956); Burke and Hara, 1963). The ratio R at two frequencies is again independent of electron density;

$$\mathcal{R} = \mathcal{C}_{3/2}\left(\frac{\omega_1}{\nu_m}\right) \Big/ \mathcal{C}_{3/2}\left(\frac{\omega_2}{\nu_m}\right) \tag{14}$$

When the collision frequency is low ($\nu_m \ll \omega$) we may use the approximation

$$\mathcal{R} \approx \frac{\omega_2^2}{\omega_1^2} \frac{\left[1 - \frac{35}{4}\frac{\nu_m^2}{\omega_1^2} + \dots\right]}{\left[1 - \frac{35}{4}\frac{\nu_m^2}{\omega_2^2} + \dots\right]} \tag{15}$$

This is equivalent to the Appleton-Hartree formula, provided we take $\nu_m^{\ 2} = \frac{4\nu^2}{35}$. We can easily compute the ratio R as a function of ν_m for any ω_1 and ω_2. This has been done for the DRTE probe for which $\omega_1 = 2.83 \times 10^7$ and $\omega_2 = 7.87 \times 10^7$, $\omega_2 = 2.78\omega_1$ and the result is shown in Fig. 2. The Appleton-Hartree result is also shown, and the difference can be seen to be appreciable. Heights in the ionosphere corresponding to the collision frequency profile adopted by Belrose are shown on the figure. It can be seen that (provided the frequency shifts are measureable) the collision frequency in the lower D-region may be determined from the experimentally measured ratio R. With electron densities of $10^2/\mathrm{cm}^3$ at 60 km, which are observed by Belrose, the frequency shifts are of the order of 100 cps. This small shift was not measurable with the present form of the probe, and the best that may be expected is confirmation of the order of magnitude of ν near 70 km. However, the experience gained shows the main shortcoming to be the data handling procedure used, and improvements can readily be made to permit useful measurements of both N and ν down to 55 or 60 km. A slightly lower value of f_1 would improve the performance, while still keeping the gyromagnetic influence small. Calculations for f_1 = 3 mc/s and f_2 = 12 mc/s are shown in Fig. 3. The ratio R in this case is a strong function of ν over the range of values from 2×10^6 to 5×10^7 sec^{-1}, corresponding to ionospheric heights from 75 km down to 55 km.

The neglect of the magnetic field is justified for two reasons. Firstly, even the lower operating frequency is at least two or three times the gyro frequency, and the effect of the field is therefore small, and is especially so in the presence of collisions as shown by the theory. Secondly, the probe determines some sort of average, not readily established, over the different directions of the field; this average will be closer to the zero field value than the value for either magneto-ionic mode.

5. Sampling Volume

An estimate of the sampling volume of the spherical capacitance probe may be made by considering the case in which the sphere is surrounded by an ion sheath, devoid of electrons, of thickness h (Fig. 4). Inside the sheath $\varepsilon = 1$, and beyond the sheath ε takes the value characteristic of the medium. The potential V of the sphere with a charge q on it is obtained by integrating the work done on a unit charge in bringing it from infinity to the sphere.

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\epsilon}\int_{\infty}^{R+h}\frac{dr}{r^2} + \int_{R+h}^{R}\frac{dr}{r^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\epsilon(R+h)} - \frac{1}{(R+h)} + \frac{1}{R}\right]$$

$$C = \frac{q}{V} = 4\pi\epsilon\epsilon_0 R\cdot\left(\frac{R+h}{R+\epsilon h}\right) \qquad (16)$$

When $\varepsilon = 1$, C reduces to $4\pi\varepsilon_o R$, the capacity in free space. When $h = o$, the correct formula for the capacity of a sphere in a dielectric medium is obtained. In the presence of a sheath a correction factor must be applied which is a function of both the relative sheath thickness and the dielectric constant.

When the capacity of the sphere is taken to be a measure of ε, the effective values are related to the real values as follows.

$$4\pi\epsilon_{eff}\epsilon_0 R = 4\pi\epsilon\epsilon_0 R\cdot\left(\frac{R+h}{R+\epsilon h}\right)$$

$$\therefore \quad \epsilon_{eff} = \epsilon\left(\frac{R+h}{R+\epsilon h}\right) \qquad (17)$$

In a weak plasma the effective electron density is then approximately related to the correct value by:

$$N_{eff} = N\cdot\left(\frac{R}{R+h}\right) \qquad (18)$$

This relation is plotted in Fig. 5. The relative error is related simply to the sheath thickness relative to the probe dimension, and is otherwise independent of the actual electron density. When h = R the apparent electron density is one half the actual value. Throughout most of the ionosphere the Debye length is not over one centimeter, and therefore the ion sheath is not a serious problem for a probe with dimensions of ten centimeters or more, as would be the case in practice.

It may therefore be concluded that the sampling volume extends out a distance of the order of the probe dimension.

6. The DRTE Probe

The operation of the DRTE probe is indicated by the block diagram (Fig. 6). Two frequencies, 4.5 and 12.0 cm/s, are each chosen for 40 msecs in turn by a relay across part of the tuning coil providing so-called low-frequency and high-frequency modes of operation. Two mixer stages lead to an output signal of near zero frequency in free space in either mode of operation when the circuitry is properly adjusted. As the probe penetrates the plasma, the tip oscillator frequency increases. A total increase of 2 mc/s is within the bandwidth capabilities of the mixers, filters, and coder. This variation permits the measurement of electron densities up to $4 \times 10^5/cm^3$ with the 4.5 mc/s oscillator, and over $10^6/cm^3$ with the 12.0 mc/s oscillator. Great care was taken to achieve good circuit stability in order to permit measurement of low electron densities.

During the 40 milliseconds of operation in either mode, a bias voltage on the tip is swept from -3 to +3 volts with respect to the rocket. As an additional part of the program of operation, a constant bias of about -100 volts is applied during every eithth 40 ms period in each mode of operation.

7. Operation During Rocket Flights

This probe was flown on two Black Brant rockets at the Churchill Research Range, Canada, during Spring 1963. The first rocket BBII-22 was fired at 1431 hours CST, 7 May during undisturbed ionospheric conditions; the second, at 2305 hours 10 June during an auroral radio wave absorption event. Two more flights are planned for Autumn 1963. On both of the spring flights the lower frequency mode provided good results in the D region above 70 or 75 km, but the high frequency mode was somewhat erratic. Electron density profiles have been deduced and are discussed in a companion paper.

A perplexing feature of both flights was a large variation of the tip oscillator frequency above 100 kms (in both modes) at the spin rate of the rocket; there must accordingly be some significant departure from cylindrical symmetry about the rocket spin axis, but the nature of this assymmetry has not been established.

While the results from the actual flights are still of a preliminary nature, they do verify that the present approach is satisfactory.

During each 40 ms period of operation the oscillator frequency increased with the bias voltage on the tip until the tip was slightly positive with respect to the rocket. With further increase in bias the frequency remained constant. Since the ratio of electron to ion mobility is at least 170 and the collection area ratio of the nose-cone to the tip was only 25, it may be assumed that the tip always remained slightly negative with respect to the plasma. The frequency during the time it was independent of bias was taken as the measure of electron density in the plasma; the ion sheath during this period may be assumed to be nearly equal to the Debye length $h_D = 6.9\sqrt{\frac{T}{n}}$, and a correction may then be applied as in Fig. 5 to obtain the actual electron density.

The frequency shift in the low frequency mode for BBII-21 is plotted as the upper curve in Fig. 7. In the D-region the shift is only a few hundred cycles per second, and in fact shifts of only a few cycles per second would need to be measured in order to determine the electron density down to 55 - 60 km. Such a small shift requires a high instrumental stability, of the order of 1 part in 10^6. Ordinarily this would not be attainable for the duration of the flight as drifts as high as 1 part in 10^2 may occur due to changes in the temperature of the circuit components, and particularly of the nose-tip itself. However, the stability requirement may be reduced to short-term effects only by the use of the high negative bias on the tip, as shown by the middle curve in Fig. 7. The effect of the negative bias is to repel the electrons far from the tip, and thus to bring the oscillator frequency to near the free space value. The difference between these curves, especially for low values of N, is a good measure of N, and it may be obtained experimentally in about one second or even less; for such short periods a stability of 1 part in 10^6 is attainable. In the present instrument the stability approached this value, but accuracy was lost in the data coding and telemetry adopted.

The opposing effects of ion acceleration when the high negative bias is applied and of the converging geometry might reasonably lead to an approximately constant value of ion density within the sheath. The calculation on this basis provides an order of magnitude of 50 cms for the sheath thickness near the E region. The observed ratio of electron density with and without the -100 V bias is from Fig. 7 equal to 6 at 95 km; if the radius 6 cms of the base of the tip is taken as a typical dimension of the probe, equation 18 yields a sheath thickness of 30 cms, in qualitative agreement with the calculation.

The sheath thickness might be expected to vary inversely as the square root of the electron density, as in the expression for the Debye length. This dependence is in fact verified by the results shown in Fig. 7 in the height range 86 to 88 km. There the electron density changes by a factor of 4 over the short height range of 2 km (over which the collision frequency would change by only a small amount) and the sheath thickness changes by approximately a factor of 2.

While an accurate estimate of the effect of ion collisions is not easy, it may be shown qualitatively that they should increase the ion density near the probe and thereby should decrease the sheath thickness. This factor is thought to account for the difference between the actual curve obtained with the high negative bias and the dashed curve in Fig. 7 which follows the $N^{-1/2}$ law from the E region down. These results suggest that the combination of a d.c. bias with the r.f. probe may provide a convenient method for studying the properties of the ion sheath.

8. Conclusions

An RF probe operating well above the plasma and gyromagnetic frequencies has been shown to be a sensitive instrument for the measurement of D region electron density. An instrument operating at 3 Mc/s with a short term frequency stability of 1 part in 10^6 would permit measurements of electron density as low as a few electrons per cm^3 in the absence of collisions. The sensitivity is decreased by a factor of 8 at 50 km because of the effect of collisions, but densities as low as a few tens of electrons per cm^3 are still detectable.

The electron density and collision frequency may be regarded as two independent variables, and their simultaneous determination requires two independent measurements. Two suitable measurements are the values of the dielectric constant at two different frequencies. In fact the ratio of the frequency shifts obtained using this form of probe operating at two frequencies is dependent only on the collision frequency and not on the density. With a choice of 3Mc/s and 12Mc/s the collision frequency can be measured accurately in the D-region between 50 and 80 kms. If high accuracy in the frequency shift measurements is completely preserved in the telemetry, then collision frequencies as low as 5×10^4 sec^{-1} characteristic of the E region at 100 km have a detectable effect.

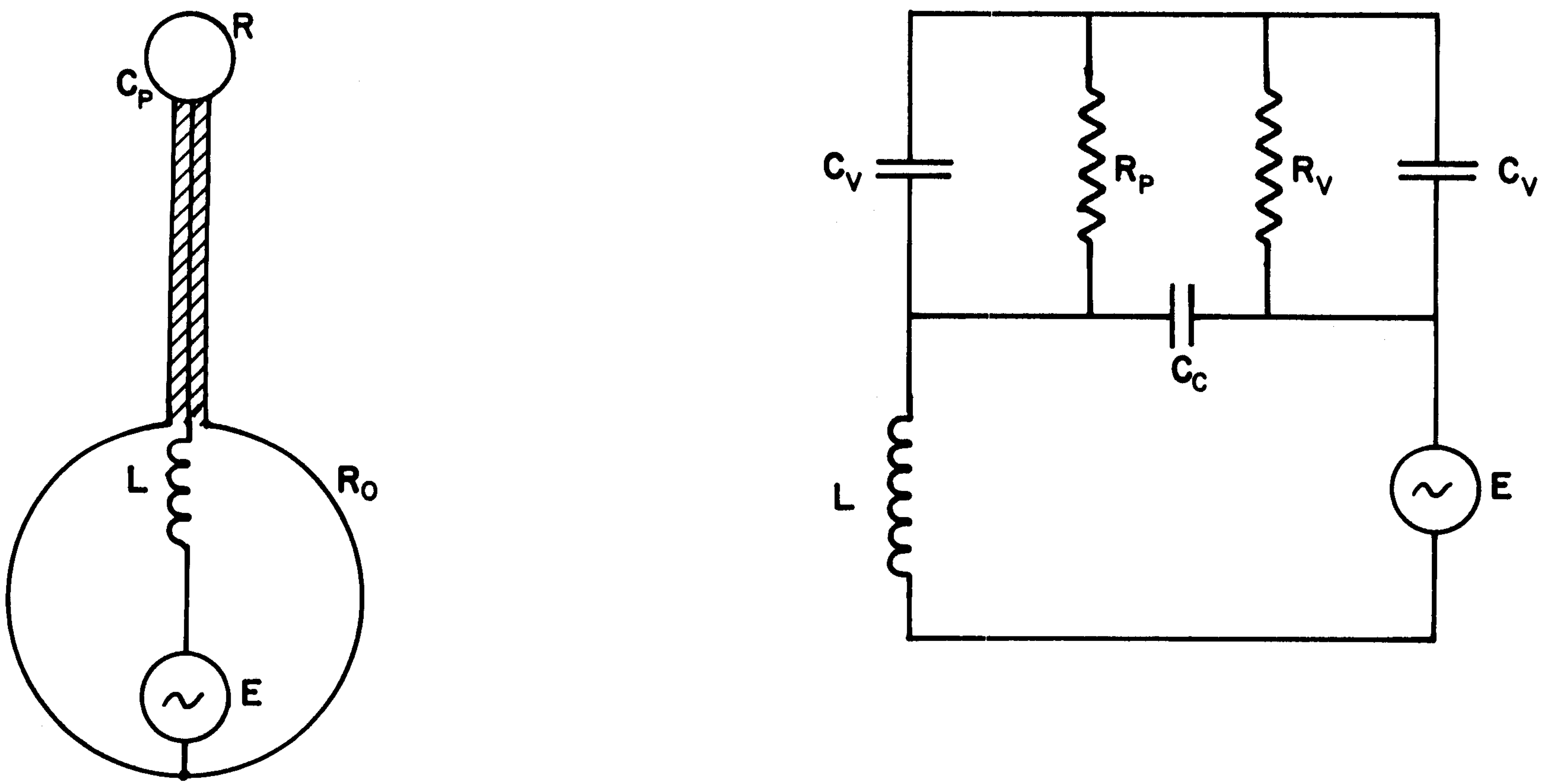

FIG. 1. SIMPLIFIED PROBE AND CIRCUIT DIAGRAM

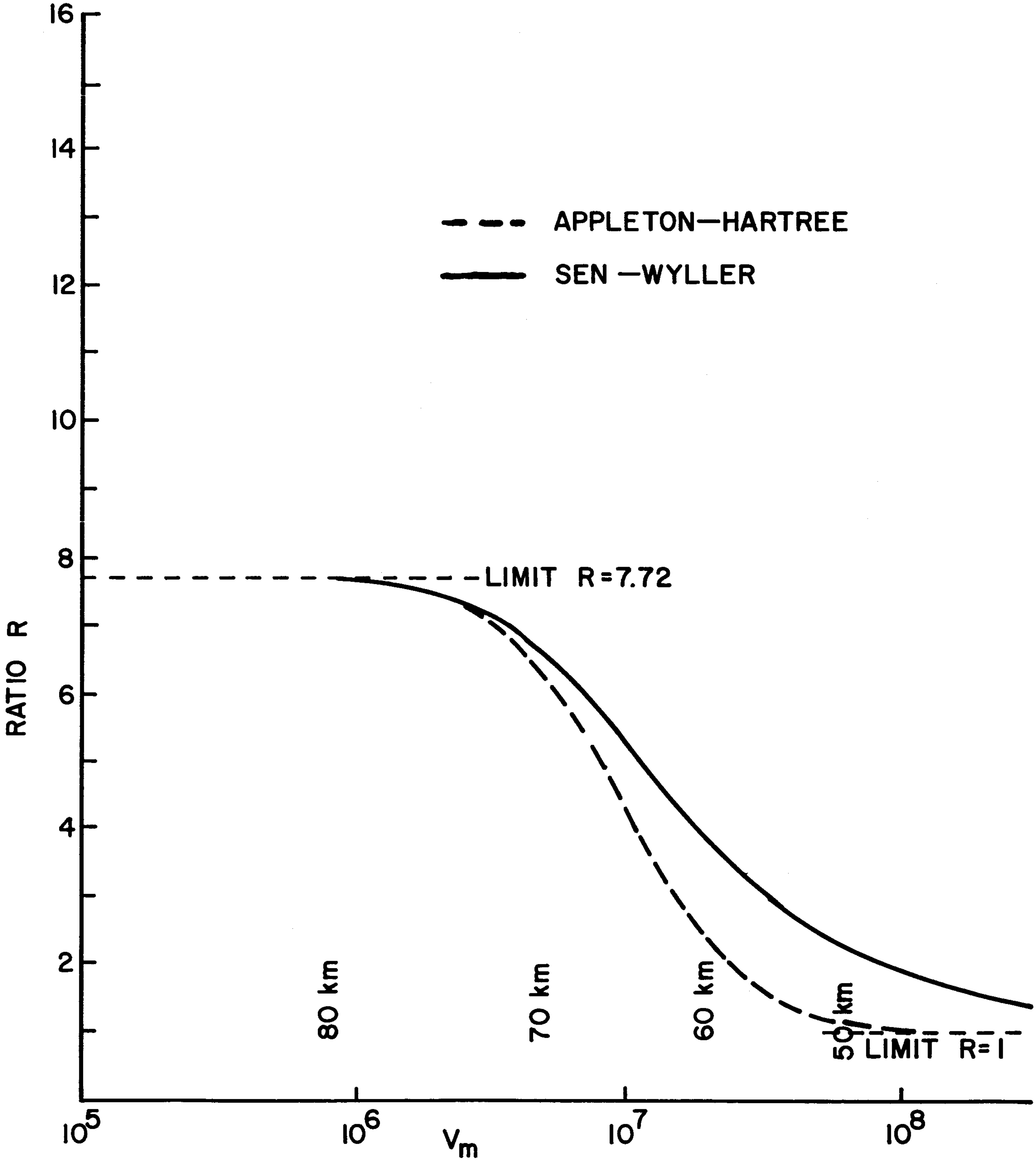

FIG. 2. RATIO R OF RELATIVE FREQUENCY CHANGES AT 4.5 Mc/s & 12.5 Mc/s AS A FUNCTION OF THE COLLISION FREQUENCY.

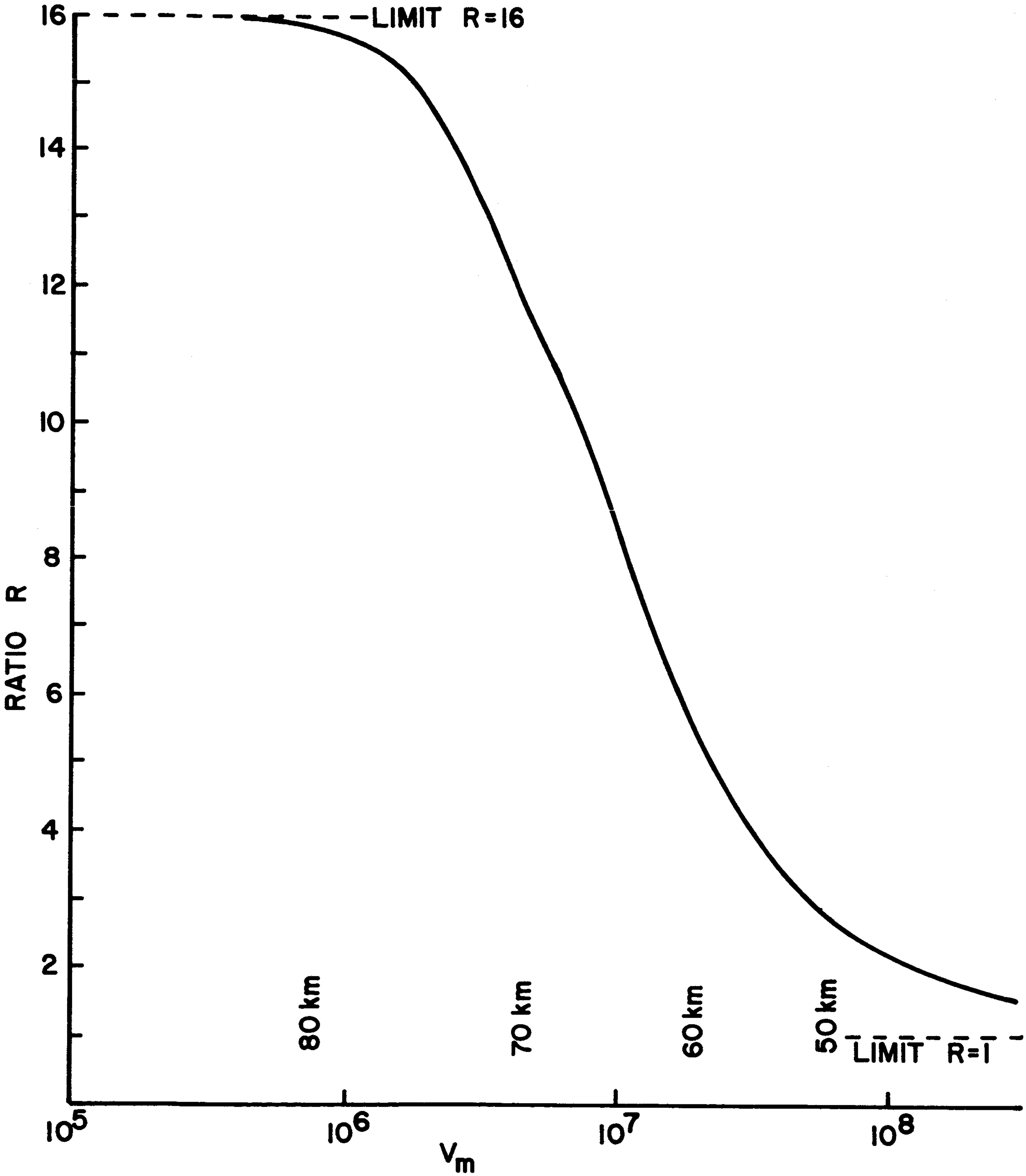

FIG. 3. RATIO R OF RELATIVE FREQUENCY SHIFTS AT 3.0 Mc/s & 12 Mc/s AS A FUNCTION OF V_m THE COLLISION FREQUENCY IN THE GENERALIZED THEORY.

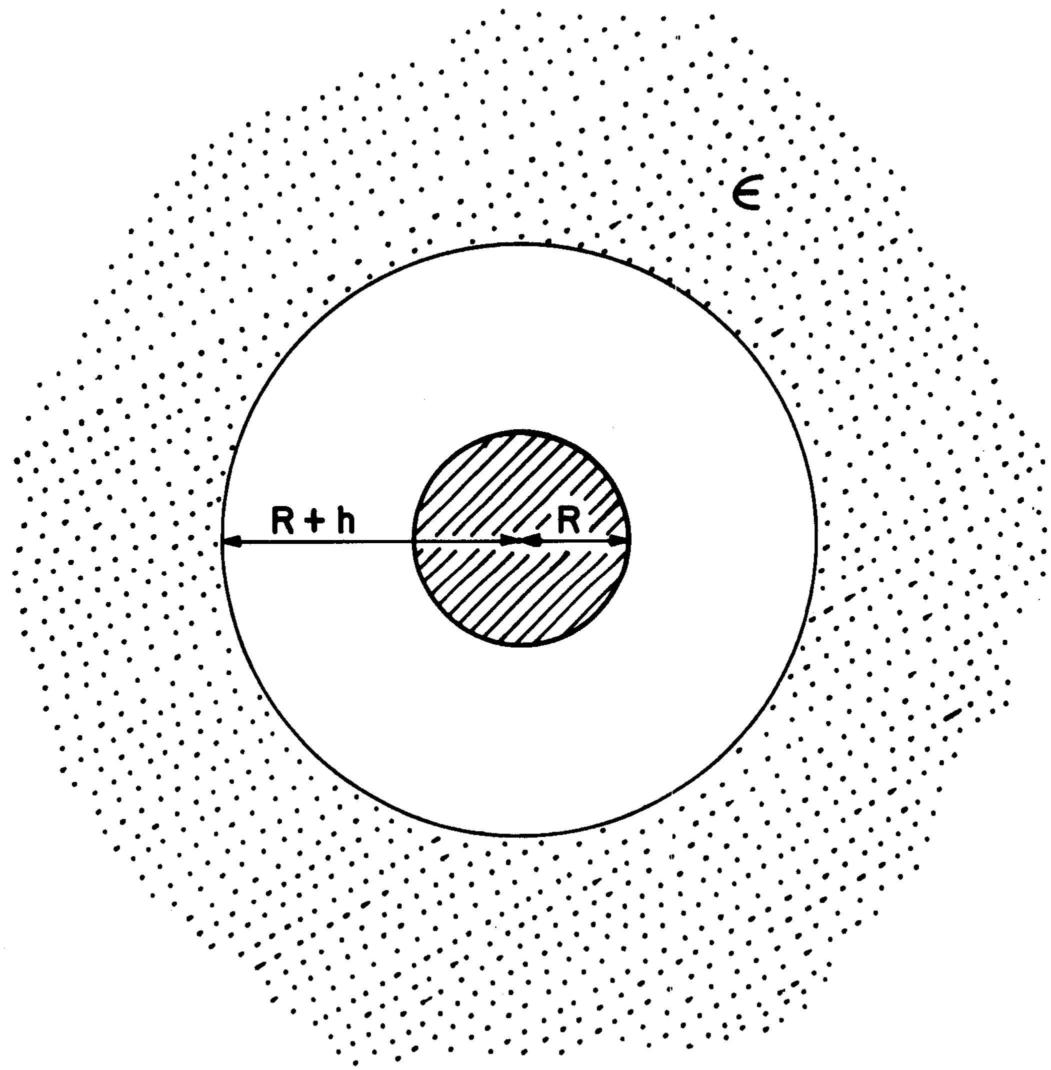

FIG. 4. MODEL OF ION SHEATH SURROUNDING PROBE

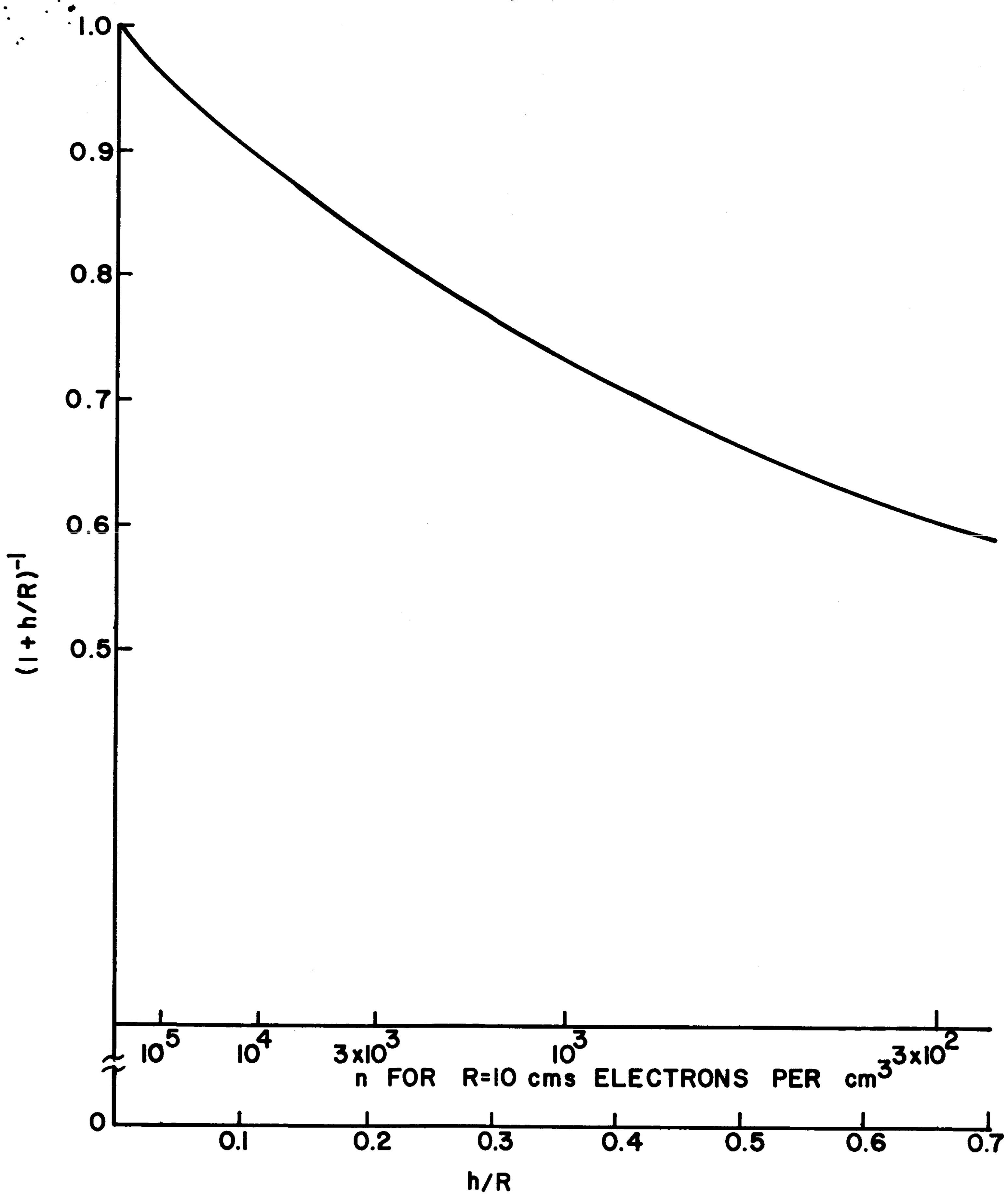

FIG.5. RATIO OF APPARENT TO ACTUAL ELECTRON DENSITY vs ION SHEATH THICKNESS OR ELECTRON DENSITY.

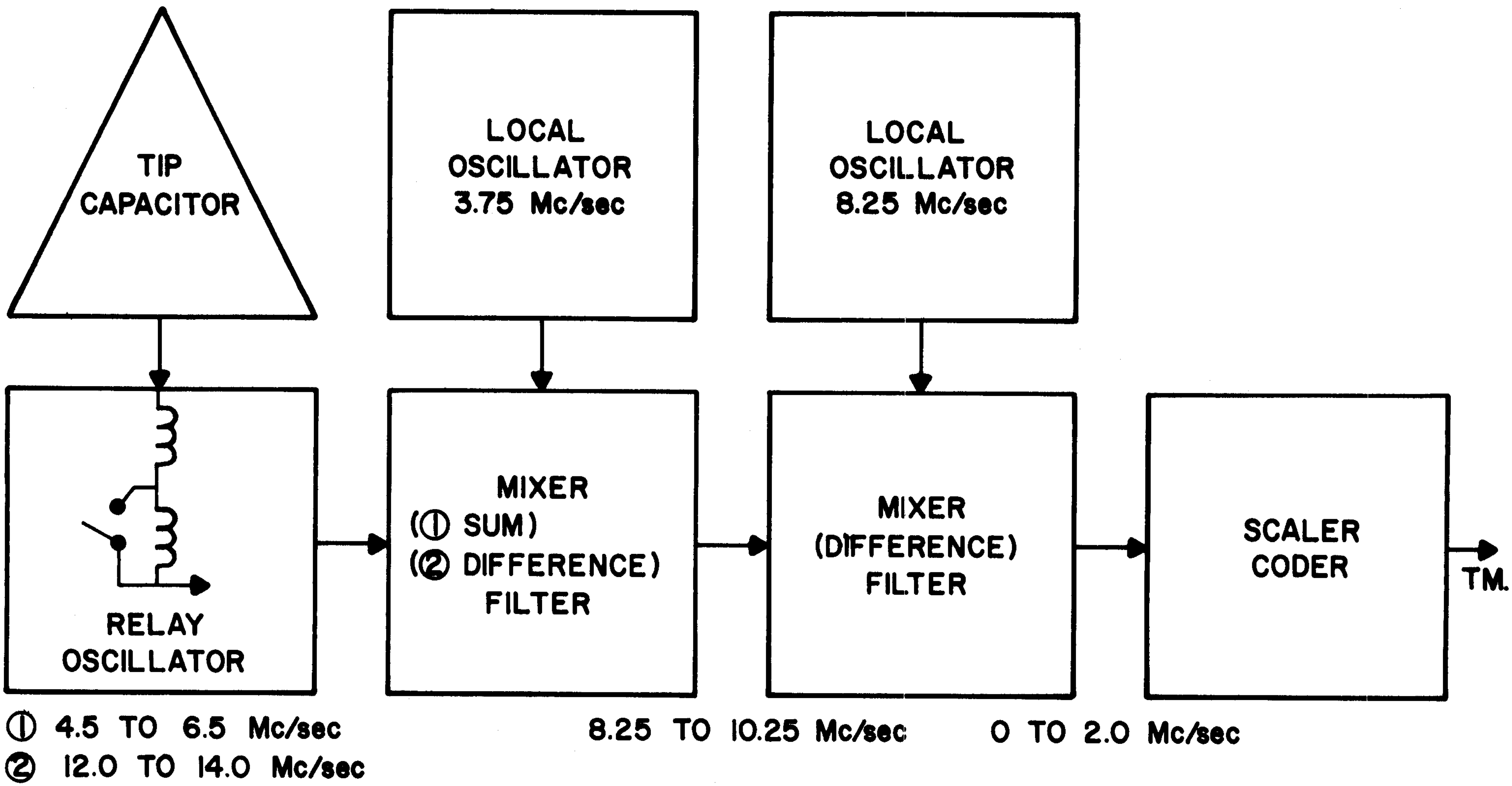

FIG. 6.
BLOCK DIAGRAM SHOWING THE OPERATION OF THE RF PROBE

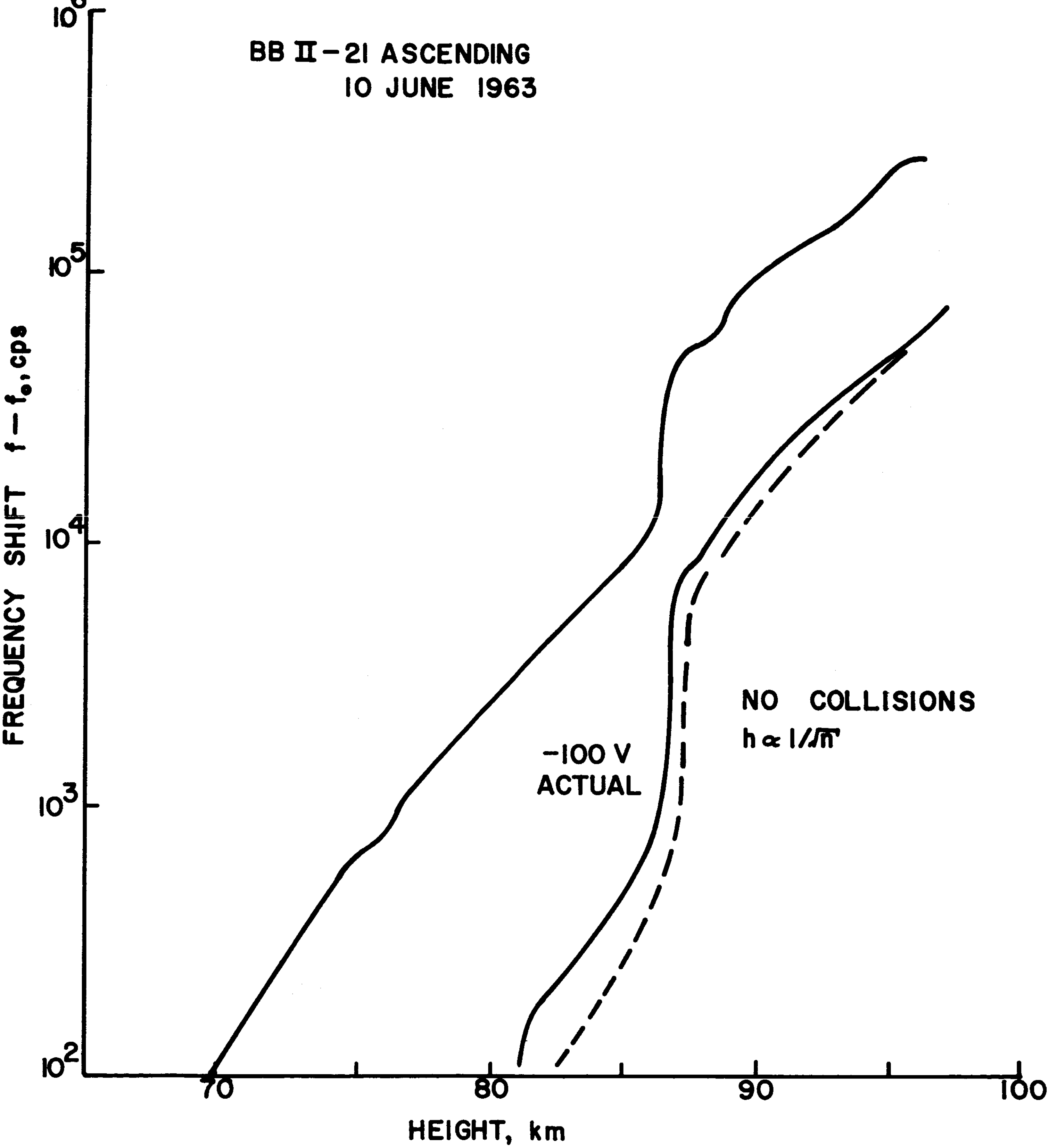

FIG. 7. FREQUENCY SHIFT OF 4.5 Mc/s OSCILLATOR DUE TO THE EFFECT OF THE PLASMA.

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